A Book on the fortextbook Option

The Student Edition

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Contents

Introduction: Student Solution Manual

This document contains the solutions to the odd-numbered problems from the historic textbook *A Book on the fortextbook Option*, by Dr. D. P. Story, *et al*.

In addition to any commented text in the source file of this document, a description of how to create a solution manual may be found in **Chapter 3**, page 37, of *A Book on the fortextbook Option*. You can learn much from reading the source file of this document as well.

You can write introductory text to the **Student Edition** and the **Instructor Edition** in the same source file by using the switch(es) `\ifisinstred` and `\ifisstudented`. If it is not compiled for the instructor edition, then it is compiled for the student edition, so normally, both of these are not needed.

The file itself was compiled using the options

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\textbookOpts{studented,lsols}
```

found in the preamble of this document.

Chapter 1

The New eqexam

1.1 Setting the page layout

1. (a) We have, $3x + 5 = 1 \implies 3x = -4 \implies \boxed{x = -\frac{4}{3}}$

(b) We have

$$\begin{aligned} \frac{1}{2}(x+5) &= \frac{1}{3}(2x-1) \\ 3(x+5) &= 2(2x-1) \\ 3x + 15 &= 4x - 2 \\ x &= 17 \end{aligned}$$

The solution is $\boxed{x = 17}$

(c) We first note that $x \neq 2$. We now solve the equation.

$$\begin{aligned} \frac{x}{x-2} + 3 &= \frac{2}{x-2} \\ x + 3(x-2) &= 2 \\ 4x - 6 &= 2 \\ x &= 2 \end{aligned}$$

Since the solution, $x = 2$ is not in the domain of the equation, we conclude that the equation has $\boxed{\text{no solution}}$, or the solution set is $\boxed{S = \emptyset}$

(d) We multiply both sides by 21.

$$\begin{aligned} \frac{x+1}{3} + \frac{x+2}{7} &= 2 \\ 7(x+1) + 3(x+2) &= 42 \\ 10x + 13 &= 42 \\ 10x &= 29 \\ x &= \frac{29}{10} \end{aligned}$$

The solution is $\boxed{x = 29/10}$

3. We use standard methods.

$$\begin{aligned} x^2 - 3x + 1 &= 0 \\ x^2 - 3x &= -1 \\ x^2 - 3x + \frac{9}{4} &= -1 + \frac{9}{4} \\ (x - \frac{3}{2})^2 &= \frac{5}{4} \\ x - \frac{3}{2} &= \pm\frac{\sqrt{5}}{2} \\ x &= \frac{3}{2} \pm \frac{\sqrt{5}}{2} \end{aligned}$$

The solutions are $\boxed{x = \frac{3}{2} - \frac{\sqrt{5}}{2}, \frac{3}{2} + \frac{\sqrt{5}}{2}}$

5. We isolate the root on the LHS and square.

$$\begin{aligned} \sqrt{x-1} + 7 &= x \\ \sqrt{x-1} &= x - 7 \\ x - 1 &= (x-7)^2 \\ x - 1 &= x^2 - 14x + 49 \\ x^2 - 15x + 50 &= 0 \\ (x-5)(x-10) &= 0 \end{aligned}$$

The results are $x = 5,\ 10$, but $x = 5$ is an extraneous solution, so the solution set is $\boxed{\{10\}}$.

7. (a) A simple linear inequality:

$$\begin{aligned} &\frac{1}{3}x - 2 \geq \frac{1}{2}x + 1 \\ &\implies 2x - 12 \geq 3x + 6 \\ &\implies -x \geq 18 \\ &\implies x \leq -18 \end{aligned}$$

The solution set is $\boxed{S = (-\infty, -18\,]}$

(b) A compound linear inequality,

$$\begin{aligned} &-1 \leq \frac{3-5x}{2} \leq 9 \\ &\implies -2 \leq 3 - 5x \leq 18 \\ &\implies -5 \leq -5x \leq 15 \\ &\implies 1 \geq x \geq -3 \quad \text{or} \quad -3 \leq x \leq 1 \end{aligned}$$

The solution set is $\boxed{S = [\,-3, 1\,]}$

9. We first find how much was invested in each account. Let x = amt invested at 6%, so $4900 - x$ was invested at 8%. The problem states that

$$.06x = .08(4900 - x) \implies .06x = 392 - .08x \implies 0.14x = 392 \implies x = 2800.$$

So, Mr. Gilg invested \$2800 at 6% and earned \$168 (\$168 = .06 · \$2800). The interest earned for the other account was the same as the first, so he earned \$168 there too. The total interest earned then by the famous Mr. Gilg is $\boxed{I = \$168 + \$168 = \$336}$

1.2 Another Section

1. (a) We use the distance formula:

$$d(P,Q) = \sqrt{(2+4)^2 + (-3-2)^2} = \sqrt{61}$$

(b) We use the midpoint formula:

$$M = \left(\frac{-4+2}{2}, \frac{2+(-3)}{2}\right) = \left(-1, -\frac{1}{2}\right)$$

3. We complete the square:

$$\begin{aligned} x^2 + y^2 - 4x + 12y = -1 &\implies (x^2 - 4x + 4) + (y^2 + 12y + 36) = -1 + 4 + 36 \\ &\implies \boxed{(x-2)^2 + (y+6)^2 = 39} \end{aligned}$$

5. (a) We have, $f(2) = \dfrac{2}{2^2+1} = \dfrac{2}{5}$

(b) We have, $f(-3) = \dfrac{-3}{(-3)^2+1} = \dfrac{-3}{9+1} = -\dfrac{3}{10}$

(c) We have, $f(2x) = \dfrac{2x}{(2x)^2+1} = \dfrac{2x}{2x^2+1}$

(d) We perform the usual tests.

$$f(-x) = \frac{-x}{(-x)^2+1} = -\frac{x}{x^2+1} = -f(x)$$

The function is odd.

Chapter 2

The fortextbook option

2.1 Building a sound foundation

1. (a) We have

$$(fg)(-2) = f(-2)g(-2) = (-5)(3) = -15$$

(b) We have

$$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{2x^2 - 5}{4x + 3}$$

(c) Composing, $(f \circ f)(x) = f(f(x)) = f(4x+3) = 4(4x+3) + 3 = 16x + 15$

(d) Composing, $(f \circ g)(x) = f(g(x)) = f(2x^2 - 5) = 4(2x^2 - 5) + 3 = 8x^2 - 17$

3. Let f be an invertible function. Suppose $f(-2) = 17$. Find $f^{-1}(17) = \boxed{-2}$; consequently, we have $(f^{-1} \circ f)(-2) = \boxed{-2}$.

5. $f(x) = a(x-h)^2 + k$ is the standard form. With the vertex information, we have $f(x) = a(x-2)^2$, we need only find the value of a. For that we use the point P: $3 = a(4-2)^2 \implies 4a = 3 \implies a = 3/4$. The final form for the function is $f(x) = \frac{3}{4}(x-2)^2$

7. We use the vertex formula, $h = -b/(2a) = -(-1)/2 = 1/2$. A **minimum** occurs since the leading coefficient is positive, which means the parabola opens up, the vertex is a minimum.

9. List the *horizontal asymptotes* (H.A.) and the *vertical asymptotes* (V.A.) of the rational function below, and label each vertical asymptote as *even* or *odd*.

$$f(x) = \frac{2x^4 - 3x^2}{(x+3)^2(x^2-4)}$$

H.A.: $\boxed{y = 2}$ V.A.: $\boxed{x = -3 \text{ (even)}, x = -2 \text{ (odd)}, x = 2 \text{ (odd)}}$

2.2 Another awesome section

1. (a) $f(x) = 3^{1-x}$, $f(3.2) = 0.089$

(b) $f(x) = e^{x/2}$, $f(4.2) = 8.166$

(c) $f(x) = -\left(\frac{1}{2}\right)^{x+1}$, $f(-3.5) = -5.657$

3. Convert $81^{1/2} = 9$ into a logarithmic form. It is apparent that, $\log_{81}(9) = 1/2$

5. We require $x - 4 > 0$ or that $x > 4$.

7. (a) $\log_4(16x^8) = \boxed{2 + 8\log_4(x)}$

We use the properties of logarithms:

$$\log_4(16x^8) = \log_4(16) + 8\log_4(x) = \log_4(4^2) + 8\log_4(x) = \boxed{2 + 8\log_4(x)}$$

(b) We have,

$$\log\left(\sqrt{\frac{x}{4}}\right) = \log\left(\frac{x}{4}\right)^{1/2} = \frac{1}{2}\log\left(\frac{x}{4}\right) = \boxed{\frac{1}{2}(\log(x) - \log(4))}$$

(c) We use the properties of logarithms:

$$\log\frac{x(x-1)^4}{(x+1)^3} = \log(x(x-1)^4) - \log(x+1)^3 = \log(x) + \log(x-1)^4 - 3\log(x+1)$$
$$= \boxed{\log(x) + 4\log(x-1) - 3\log(x+1)}$$

9. (a) $\log(310.4) \approx \boxed{2.4919}$

(b) $\ln(310.4) \approx \boxed{5.7379}$

(c) $\log_3(11.4) \approx \boxed{2.2172}$

(d) $\log_{1/2}(11.4) \approx \boxed{-3.5110}$

11. (a) $\log_5(2x-1) = 1.1 \implies 2x - 1 = 5^{1.1} \implies \boxed{x = \dfrac{1 + 5^{1.1}}{2} \approx 3.4365}$

(b) We have,

$$\log_2(x-1) - \log_2(x-2) = 3 \implies \log_2\frac{x-1}{x-2} = 3 \implies \frac{x-1}{x-2} = 2^3$$
$$\implies x - 1 = 8(x-2) \implies x - 1 = 8x - 16 \implies 15 = 7x \implies \boxed{x = \frac{15}{7} \approx 2.1429}$$

2.3 One more time!

1. Recall the point $(\cos(x), \sin(x))$ lies on the unit circle, hence, the point satisfies the equation $\sin^2(x) + \cos^2(x) = 1$.

3. If we divide both sides of $\sin^2(x) + \cos^2(x) = 1$ by $\sin^2(x)$, we obtain the identity $1 + \cot^2(x) + 1 = \csc^2(x)$.

5. Take the addition formula for the cosine function,

$$\cos(x + y) = \sin(x)\sin(y) - \cos(x)\cos(y)$$

and put $x = y$ to obtain the basic equation, $\cos(2x) = \cos^2(x) - \sin^2(x)$. Now substitute $\sin^2(x) = 1 - \cos^2(x)$ into this equation to obtain $\cos(2x) = 2\cos^2(x) - 1$; substitute $\cos^2(x) = 1 - \sin^2(x)$ into the first equation, we obtain the last variation, $\cos(2x) = 1 - 2\sin^2(x)$

7. $\sinh(x) = (e^x - e^{-x})/2$

2.4 Once more, once!

1. **(a)** The function $f(x) = (4.3)^x$ is an exponential function with a base of $a =$ __4.3__ .
   (b) __T__ (T or F) One of the properties of logarithms is $\log_a(x) - \log_a(y) = \log_a(x/y)$
   (c) The correct alternative is , the domain of $f(x) = \log_a(x)$ is $\mathrm{Dom}(f) =$.
   (d) The inverse of the function $f(x) = 7^x$ is $f^{-1}(x) =$ $\log_7(x)$____.

Demo Problem Set

1. Solve the equation $2x + 5 = -2$ for x. Indeed,

$$\begin{aligned} 2x + 5 &= -2 && \text{given} \\ 2x &= -7 && \text{subtract 5 both sides} \\ x &= \boxed{-\frac{7}{2}} && \text{divide by 3 both sides} \end{aligned}$$

2.6 Review Exercises

Section 2.2

1. **(a)** We have

$$\begin{aligned} \frac{1}{2}(x+5) &= \frac{1}{3}(2x-1) \\ 3(x+5) &= 2(2x-1) \\ 3x + 15 &= 4x - 2 \\ x &= 17 \end{aligned}$$

   The solution is $\boxed{x = 17}$
   (b) We have,

$$6x + 5 = 3x + 1 \implies 3x = -4 \implies \boxed{x = -\frac{4}{3}}$$

3. **(a)** A simple linear inequality:

$$\begin{aligned} \frac{1}{3}x - 2 &\geq \frac{1}{2}x + 1 \\ &\implies 2x - 12 \geq 3x + 6 \\ &\implies -x \geq 18 \\ &\implies x \leq -18 \end{aligned}$$

   The solution set is $\boxed{S = (-\infty, -18\,]}$
   (b) We use standard techniques,

$$\begin{aligned} |x - 4| &\leq 6 \\ -6 \leq x &- 4 \leq 6 \\ -2 &\leq x \leq 10 \end{aligned}$$

   The solution is $\boxed{S = [\,-2, 10\,]}$

Section 2.3

5. (a) We use standard methods.

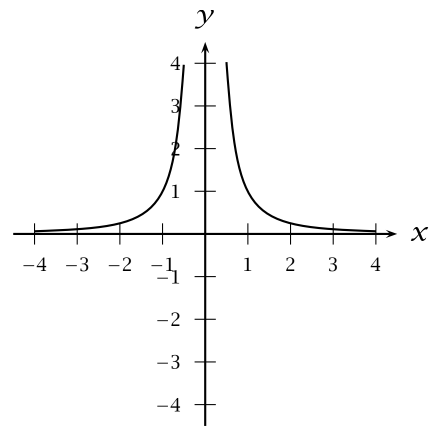
$$f(x) = 3 - 5x \implies y = 3 - 5x$$
$$\implies x = 3 - 5y \implies 5y = 3 - x$$
$$\implies y = \frac{3-x}{5} \implies \boxed{f^{-1}(x) = \frac{3-x}{5}}$$

(b) We use standard methods.

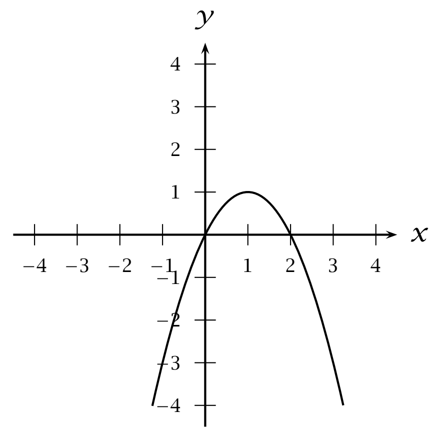
$$f(x) = 6x^3 + 2 \implies y = 6x^3 + 2$$
$$\implies x = 6y^3 + 2 \implies 6y^3 = x - 2$$
$$\implies y = \sqrt[3]{\frac{x-2}{6}} \implies \boxed{f^{-1}(x) = \sqrt[3]{\frac{x-2}{6}}}$$

Chapter 2. Chapter Quiz

1. (a) Graph $f(x) = \dfrac{1}{x^2}$.



1. (b) Graph $f(x) = 1 - (x-1)^2$.



2. (a) We have

$$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{2x^2-5}{4x+3}$$

(b) Composing,

$$\begin{aligned}(f\circ g)(x) &= f(g(x)) = f(2x^2-5)\\ &= 4(2x^2-5)+3\\ &= \boxed{8x^2-17}\end{aligned}$$

3. For a polynomial of degree 17, according to theory, the maximum number of zeros is __17__ , and the maximum number of turning points is __16__ .

4. Removing the functional notation, we have, $y = 3x+2$. Interchange the roles of x and y, and solve for y. We have...

$$\begin{aligned} x &= 3y+2 && \text{interchange } x \text{ and } y\\ 3y &= x-2 && \text{subtract 2, both sides}\\ y &= \frac{x-2}{3} && \text{divide by 3}\\ g^{-1}(x) &= \frac{x-2}{3} && \text{use functional notation}\end{aligned}$$

The domain is $(-\infty,\infty)$.

5. We use the vertex formula, $h = -b/(2a) = -(-8)/4 = 2$, and so $h = f(2) = 8-16+5 = -3$. The coordinates of the vertex is then $V(2,-3)$. Because the coefficient of x^2 is positive, the parabola opens up, which implies the vertex is a *minimum.*

6. We have $y = \dfrac{k}{x}$, but $8 = \dfrac{k}{4} \implies k = 32$. thus, $y = \dfrac{32}{x}$.

7. (a) We use the properties of logarithms:

$$\begin{aligned}\log\frac{x(x-1)^4}{(x+1)^3} &= \log(x(x-1)^4) - \log(x+1)^3 = \log(x) + \log(x-1)^4 - 3\log(x+1)\\ &= \boxed{\log(x) + 4\log(x-1) - 3\log(x+1)}\end{aligned}$$

(b) We use the properties of logarithms.

$$\frac{1}{2}(\log(x) + 3\log(y)) = \frac{1}{2}(\log(x) + \log(y^3)) = \frac{1}{2}\log(xy^3) = \log(xy^3)^{1/2}$$

8. (a) We use standard techniques, take logs of both sides, and solve for x.

$$5^{2x} = 7.3 \implies \ln 5^{2x} = \ln 7.3 \implies 2x\ln 5 = \ln 7.3 \implies \boxed{x = \frac{\ln 7.3}{2\ln 5} \approx 0.6176}$$

(b) We solve by converting log to exponential.

$$\log_5(2x-1) = 1.1 \implies 2x-1 = 5^{1.1} \implies \boxed{x = \frac{1+5^{1.1}}{2} \approx 3.4365}$$