

Instructions. (52 points) Solve each of the following questions.

- (5^{pts}) 1. An example of a fill-in question: It is well known that _____ and _____ are jointly credited as the founders of modern calculus.
- (3^{pts}_{ea.}) 2. *True or False.* No justification needed.
- (a) _____ If triangles have 4 sides, then all monkeys are green. Now is the time for all good men to come to the aid of their country.
- (b) _____ $1 + 1 = 3 \iff \sqrt{2}$ is a rational number. Now is the time for all good men to come to the aid of their country.
- (c) _____ $(\forall x)(\exists y)(xy > 1)$ (x, y real numbers). Now is the time for all good men to come to the aid of their country.
- (d) _____ $(\forall x)(\exists y)(\forall z)(z(x + y) > 0)$, (x, y , and z real numbers).
- (11^{pts}) 3. Select the correct answer for each of the following multiple choice. There is only one correct answer.
- (a) (5 pts) In what ancient year did Columbus sail the ocean blue?
- (1) 1490 (2) 1491 (3) 1492 (4) 1493
- (b) (6 pts) In what year did King John sign the Magna Carta?
- (1) 1213 (2) 1214 (3) 1215 (4) 1216
(5) 1217 (6) 1218 (7) 1219 (8) None of these
- (5^{pts}) 4. Which of the following best describes Augustin Cauchy?
- | | |
|---|---|
| <input type="checkbox"/> He developed the Calculus while his University was closed for the plague. | <input type="checkbox"/> He first formulated a precise definition of the limit and continuity of a function. |
| <input type="checkbox"/> Given credit for first using the functional notation $f(x)$. | <input type="checkbox"/> Gave a rigorous definition of the definite integral—an integral that now bears his name. |
| <input type="checkbox"/> He created the “bell-shaped curve” and first used the method of least squares. | <input type="checkbox"/> His notation for the derivative and the integral is used even to this day. |
- (5^{pts}) 5. Define a function by $h(x) = \int_{x^2}^5 \sqrt{t^2 + 4} dt$. Calculate $h'(x)$.
- $h'(x) =$
- (7^{pts}) 6. Consider the region bounded by the curves $x = y^{3/2}$, $y = 4$, and the y -axis. The region is rotated about the y -axis, calculate the volume of this solid of revolution.

- (7^{pts}) 7. The base of a solid S is bounded by the x -axis, the line $y = 2x$ and the line $x = 2$. Each cross-section perpendicular to the x -axis is a *rectangle*. The base of each rectangle lies on the base of the solid and the height of the rectangle is twice that of the base. Find the volume of this solid S . (*Hint*: Read the description of the solid carefully, draw a picture of the base of the solid S , draw a typical cross-section, and compute the lengths of the sides of the rectangle as a function of x .)

Recall: Use the following facts freely throughout the exam.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad R = 1, \quad \sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}, \quad R = \infty,$$
$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n, \quad R = \infty, \quad \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}, \quad R = \infty$$